PHYC 581: High Energy Astrophysics
Fall 2018

Lec 19: 10/24/2018

Formation of a Disk:

Once matter flows across the Lagrange point L_1, it will sweep past the compact object in its Roche lobe. The plasma at L_1 has an angular momentum with respect to the primary star. It therefore forms an orbiting ring about the accretor due to dissipation of energy.

The velocity of mass flowing across the point L_1 has both parallel and perpendicular components ($v_{\parallel}$ and $v_{\perp}$ respectively). They are given by:

$v_{\perp} \sim b_1 \omega$, $v_{\parallel} \sim c_s$

The parallel component is due to thermal random motion. For typical stellar envelope temperatures $T \lesssim 10^5$ K, we have:

$v_{\parallel} \sim 10 \text{ km s}^{-1}$

After using the Kepler's third law, we find:

$$v_1 \sim 100 \left(\frac{M}{M_\odot}\right)^{\frac{1}{3}} (1+q)^{\frac{1}{3}} \left(\frac{P_{orb}}{1\,day}\right)^{-\frac{1}{3}} \; km\,s^{-1}$$

Here, $\omega = \frac{2\pi}{P_{orb}}$. Within the primary's Roche lobe, the infalling mass is controlled by M_1 potential. As a result of energy dissipation, the orbit circularizes at a radius R_{circ} where the Keplerian angular momentum is equal to the initial angular momentum of the mass at L_1. Thus:

$$v_\phi(R_{circ}) = \left(\frac{GM_1}{R_{circ}}\right)^{1/2}$$

$$R_{circ}\, v_\phi(R_{circ}) = b_1^2 \omega$$

Together with the Kepler's law, this results in:

$$\frac{R_{circ}}{a} = (1+q)\left(\frac{b_1}{a}\right)^4 = (1+q)\left[0.500 - 0.227 \log q\right]^4$$

For $q=1$, we find $b_1 = 0.500\,a$, $R_1 = b_1$, and $R_{circ} = 0.125\,a$.

It is seen that $R_{circ} = 0.33\, R_1$ in this case. Actually, this result is generally true as $\frac{R_{circ}}{R_1} \sim \frac{1}{3} - \frac{1}{2}$ in compact binary systems. This ensures that the infalling mass will stay in the Roche lobe of the primary.

One has to also compare R_{circ} with the radius of the primary R_*. If $R_{circ} > R_*$, then a disk will be indeed formed. Otherwise, the accreted mass will fall onto the compact object. It can be seen that for any realistic binary parameters, we have $R_* < R_{circ}$. For example, for $P_{orb} \sim 1\,h$ and $q=1$, we find $R_{circ} \sim 3.5 \times 10^9$ cm. For a compact primary, the radius R_* cannot be larger than that for a white dwarf, and hence $R_* < 10^9$ cm. This implies that the gas flowing from L_1 toward M_1 misses the primary entirely and settles into a ring-like orbit that has ___ radius $\sim R_{circ}$. After this happens, viscosity enters the game and disperses the material to form a differentially rotating disk. We will discuss the theory of accretion disk and their physical properties in detail in the next lectures. This is an important topic because in order to fully understand a high energy source, we must have a viable theory of matter rotating in Keplerian orbits about a central accretor.

Again, we would like to emphasize on the important role of dissipation in the formation of a disk. Conservation of the angular momentum and the loss of energy imply that an element of the gas will eventually settle on a keplerian orbit. Otherwise, each element of the gas would orbit along its own elliptical trajectory. An important example of this is dissipationless dark matter particles in the galactic halo.

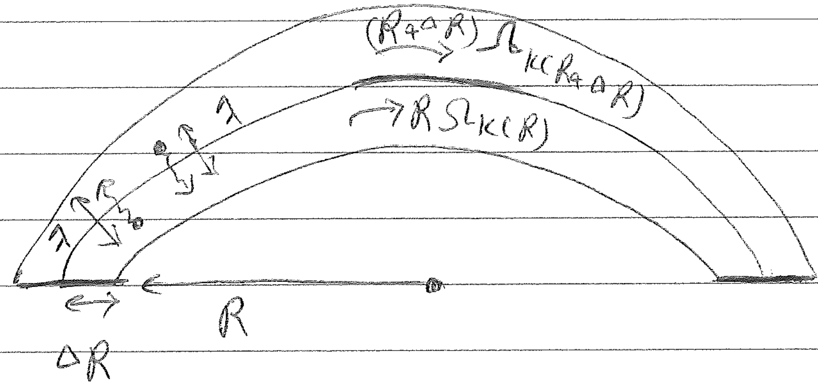
Accretion Disk Theory:

Whether a compact object accretes from the surrounding medium, or from a binary companion, the accreting plasma settles into a disk perpendicular to its net angular momentum vector. Even if the accretion proceeds spherically at first, cooling processes eventually dissipate the plasma's support in the parallel direction away.

Thin Disk Theory:

To describe the structure of the disk, we first find hydrodynamic equations similar to those derived in the case of spherical accretion. We will make full use of the specific geometry in the Keplerian motion.

Consider a geometrically thin disk rotating close to an orbital plane:



At any radius R, the matter rotates in a ring with a circular velocity $v_\phi(R) = R\,\Omega_K(R)$, where:

$$\Omega_K(R) = \left(\frac{GM}{R^3}\right)^{\frac{1}{2}}$$

This is the Keplerian angular velocity. The mass and angular momentum of the ring at radial distance R are:

Mass $= 2\pi R\,\Delta R\,\Sigma$, Angular Momentum $= (2\pi R\,\Delta R\,\Sigma)\,R^2\,\Omega_K(R)$

Here $\Sigma = \rho H$ is the surface density, with "H" being the thickness of the disk. The equation for the conservation of mass is:

$$R\,\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}\left(R\,\Sigma\,v_R\right) = 0$$

Here v_R is the radial velocity (inward) due to accretion.

The equation for the angular momentum is:

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega) + \frac{\partial}{\partial R}(R\Sigma v_R R^2\Omega) = -\frac{1}{2\pi}\frac{\partial \Gamma_{out}}{\partial R}$$

Here, Γ_{out} is the viscous torque that a ring exerts to the neighboring outer ring, which is given by:

$$\Gamma_{out} \sim 2\pi R\Sigma\tilde{v}\, R(R+\lambda)\,[\Omega(R) - \Omega(R+\lambda)]$$

λ and $\tilde{v}$ are the length scale and speed, respectively, for matter crossing between the neighboring rings. This can be due to thermal motion or turbulence. For example, in case of thermal motion, λ represents the mean-free-path of particles in the gas. By defining the coefficient of kinematic friction $\nu \equiv \lambda\tilde{v}$, we have:

$$\Gamma_{out} \simeq -2\pi\nu\Sigma R^3\Omega' \qquad \left(' \equiv \frac{\partial}{\partial R}\right)$$

After using this expression in the equation for the conservation of the mass, the angular momentum equation can be written as follows:

$$R \Sigma v_R (R^2 \Omega)' = \frac{\partial}{\partial R} (R \nu \Sigma R^2 \Omega')$$

For a Keplerian flow, $\Omega = \Omega_K = \left(\frac{GM}{R^3}\right)^{1/2}$, which results in:

$$v_R = \frac{-3}{\Sigma R^{1/2}} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2})$$

In situations where ν is constant and the disk is steady state, the mass conservation implies that $R \Sigma v_R = \text{Const.}$, and hence the accretion rate follows:

$$\dot{M} = -2\pi R \Sigma v_R = \text{Const.}$$

Also:

$$v_R \sim O\left(\frac{\nu}{R}\right) \Rightarrow \frac{v_R}{c_s} \sim \alpha\left(\frac{H}{R}\right)$$

Here, we have used the useful parametrization $\nu \equiv \alpha c_s H$.

For thin disks $H \ll R$, hence $v_R \ll c_s$, which validates the assumption of quasi-Keplerian motion of the disk. Thus, although the shear viscosity leads to outward transfer of angular momentum and inflow of mass, the dominant velocity at any given radius is alway azimuthal. As we shall see soon, by analyzing the vertical structure of thin disks, we typically have $\frac{H}{R} \lesssim O(0.01)$, which implies the disks are very thin.